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Propagation of Weak MHD Waves in Steady Hypersonic Flows with Radiation

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Introduction

THE effect of nonlinearity on wave propagation has been the subject of great interest from mathematical and physical points of view. The propagation of a one-dimensional acceleration wave and its termination into a shock wave because of nonlinear steepening has been extensively studied during the last decade by several authors.[1,2,3] Jeffrey[4] developed the idea of jump discontinuities in nonlinear hyperbolic systems of equations with two independent variables. Suhubi and Jeffrey,[5] Varley,[6] Green,[7] and Collins[8] studied the growth and decay of one-dimensional acceleration waves in a variety of materials. Recently Ram and Pandey[9] solved the problem of the breakdown of an acceleration wave in transient gas flows with vibrational relaxation. The main academic interest of the present communication is to study the effects of thermal radiation under an optically thin gas approximation[10] on the breakdown of acceleration waves in hypersonic MHD flows.

Intrinsic Properties of Wave Motion

The basic equations of a steady hypersonic MHD flow of a nongray gas in an optically thin medium can be expressed in terms of intrinsic coordinates (s,n) in the form

$$q\rho_s+\rho q_s+\rho q\theta_n=0$$

$$\rho q q_s+p_s+h_s=0$$

$$\rho q^2\theta_s+p_n+h_n=0$$

$$qh_s+2hq_s+2hq\theta_n=0$$

$$qp_s+\gamma p(q_s+q\theta_n)+4(\gamma-1)\alpha_p a_R T^4=0 \qquad (1)$$

where q, T, α_p, a_R, and θ, respectively, denote the total gas velocity, the absolute temperature, the Planck mean absorption coefficient, the Stefan-Boltzmann constant, and the angle of deflection; s and n represent the measure of distance along and normal to a streamline. A subscript will in general denote partial differentiation unless stated otherwise. For a nongray gas the Planck mean absorption coefficient α_p varies with frequency, but for a gray gas it can be treated as a constant.

The system of Eqs. (1) can be put in the form

$$U_s+AU_n+B=0 \qquad (2)$$

where U and B are column matrices and A is a square matrix of order 5. These are given by

$$U=(p\ \ q\ \ \theta\ \ h\ \ \rho)$$

$$B=\frac{4(\gamma-1)\alpha_p a_R T^4}{q(q^2-c^2-b^2)}\begin{pmatrix}\gamma p+(q^2-c^2-b^2)\\ -q\\ 0\\ 2h\\ \rho\end{pmatrix}$$

and

$$A=\begin{pmatrix}0 & 0 & \gamma p q^2/(q^2-c^2-b^2) & 0 & 0\\ 0 & 0 & -q/(q^2-c^2-b^2)(c^2+b^2)^{-1} & 0 & 0\\ 1/\rho q^2 & 0 & 0 & 1/\rho q^2 & 0\\ 0 & 0 & 2hq^2/(q^2-c^2-b^2) & 0 & 0\\ 0 & 0 & \rho q^2/(q^2-c^2-b^2) & 0 & 0\end{pmatrix}$$

Here $b=(2h/\rho)^{1/2}$ and $c=(\gamma p/\rho)^{1/2}$ are the Alféven speed and the speed of sound, respectively. The system of Eqs. (2) is quasilinear with five real characteristics. The eigenvalues of the matrix A are

$$\lambda^1=(1+M_f^2)^{1/2}(M^2-M_f^2-1)^{-1/2},$$

$$\lambda^2=-(1+M_f)^{1/2}(M^2-M_f^2-1)^{-1/2},$$

$$\lambda^3=0=\lambda^4=\lambda^5$$

and the corresponding eigenvectors are

$$L^1=\begin{pmatrix}0\\ -1/q\\ Q\\ 1/pq^2\\ -b^2/pq^2\end{pmatrix},L^2=\begin{pmatrix}0\\ -1/q\\ -Q\\ 1/pq^2\\ -b^2/pq^2\end{pmatrix},L^3=\begin{pmatrix}1\\ pq\\ 0\\ 1\\ 0\end{pmatrix}$$

$$L^4=\begin{pmatrix}0\\ 0\\ 0\\ 1\\ 0\end{pmatrix},L^5=\begin{pmatrix}0\\ 0\\ 0\\ 0\\ 1\end{pmatrix}$$

where $M=q/c$, $M_f=b/c$, and $Q=(1+M_f^2)^{1/2}(M^2-M_f^2-1)^{-1/2}$. System (2) admits discontinuities which propagate along the forward-moving characteristics. In order to study the characteristic properties of wave motion we introduce two characteristic coordinates $\phi(s,n)$ and $\psi(s,n)$ such that $\phi=$const. represents a wave front and $\psi=s$. The leading forward characteristic front can be represented by $\phi(s,n)=0$. Any flow property $f(s,n)$ is continuous across $\phi(s,n)=0$, but f_n an f_s undergo finite jumps across it. Such a wave front $\phi=0$ represents a weak discontinuity. The transformation $(s,n)\rightarrow(\phi,\psi)$ is nonsingular, provided the Jacobian of the transformation $J=n_\phi=1/\phi_n$ is nonzero and positive. Let us consider an open region R bounded by two characteristics $\phi(s,n)=0$ and $\xi(s,n)=0$ such that no characteristics issuing from the origin enter this open region R. We assume that U remains smooth in R at least for a finite time during which the transformation is nonsingular. Transforming the quasilinear system (2) into a new coordinate system (ϕ,ψ) and

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premultiplying by L^j such that $b^j = L^j B$, we get

$$L^j \{ n_\phi U_\psi + (\lambda - \lambda^1) U_\phi \} + n_\phi b^j = 0 \tag{3}$$

which provides

$$L^1 U_\psi + b^1 = 0 \text{ for } \lambda = \lambda^1 \tag{4}$$

If $C^{(\phi)}$ represents the wave front trace, the lower suffix ϕ denotes differentiation along the normal to $C^{(\phi)}$ and the lower suffix ψ denotes differentiation along $C^{(\phi)}$. The boundary conditions at the wave front $\phi = 0$ are:

(i) U is continuous: $[U] = 0$;

(ii) U_ψ is continuous: $[U_\psi] = 0$;

(iii) U_ϕ is discontinuous: $[U_\phi] \neq 0 = \pi(\psi)$;

(iv) n_ϕ is discontinuous: $[n_\phi] \neq 0 = X(\psi)$

where the bracket $[Z]$ denotes the jump in Z across a wave front $\phi = 0$, i.e., $[Z]_{\phi=0^+}^{\phi=0^-} = Z(0^-, \psi) - Z(0^+, \psi)$. From the definition of $X(\psi)$ we observe that

$$X(\psi) + (n_\phi)_{\phi=0^+} = (n_\phi)_{\phi=0^-} \tag{5}$$

If U_0 corresponds to the constant state condition ahead of the wave front, we have

$$B(U_0) = 0$$

It follows from Eq. (3) that

$$L_0^j \pi(\psi) = 0 \text{ if } \lambda \neq \lambda^1 \tag{6}$$

Differentiating Eq. (4) with respect to ϕ at any point of the open region R and using the jump relations we get

$$L_0^1 \pi_\psi(\psi) + [\nabla_u (L^1 B)]_0 = 0 \tag{7}$$

where ∇_u denotes the gradient operator with respect to the components of the vector U.

The equation of an outgoing characteristic can be written as

$$n_\psi = \lambda^1 \tag{8}$$

Differentiating Eq. (8) with respect to ϕ at any point in R, allowing this point to tend to a point on the wave front and using the jump conditions, we get

$$X_\psi = [\nabla_u (\lambda^1)]_0 \pi$$

which yields

$$X = \bar{X} + \int_0^s [\nabla_u (\lambda^1)]_0 \pi \, d\psi, \text{ with } \bar{X} = \lim_{\psi \to 0} X \tag{9}$$

We would expect the solution to break down after some finite critical distance S_c which can be determined by the intersection of two outgoing characteristics. At such a critical point the Jacobian of the transformation must vanish and therefore we have $X(\psi) + (n_\phi)_0 = 0$. Hence the critical distance S_c is given by

$$1 + \int_0^{S_c} [\nabla_u (\lambda^1)]_0 \pi / \bar{n}_\phi \, d\psi = 0 \tag{10}$$

In consequence of Eq. (6) we have

$$\pi_1 + \rho_0 q_0 \pi_2 = 0, \; q_0 Q_0 \pi_3 = -\pi_2, \; \pi_4 = \pi_5 = 0 \tag{11}$$

Also we have

$$[\nabla_u (L^1 B)]_0 \pi = \frac{4(1-\gamma)\alpha_p a_R T_0^4 (3\gamma M^2 + 1)_0}{q_0^2 c_0^2 (M^2 - M_f^2 - 1)_0} \times \{ T(\alpha_p)_T (4\alpha_p)^{-1} + 1 \}_0 \pi_2 \tag{12}$$

Using Eqs. (11) and (12) in Eq. (7), we obtain an equation for π in the form

$$\pi_{2\psi} + C_1 \pi_2 = 0 \tag{13}$$

where $C_1 = -[\nabla_u (L^1 B)]_0 q_0 \pi / 2\pi_2$ is a constant of the constant state. Thus the jump discontinuities $\pi = [U_\phi]$ across $\phi(s,n) = 0$ can be expressed as

$$\pi = \pi_2 \exp(-C_1 s) \begin{pmatrix} -\rho_0 q_0 \\ 1 \\ -(Q_0 q_0)^{-1} \\ 0 \\ 0 \end{pmatrix} \tag{14}$$

Now differentiating Eq. (8) with respect to ϕ at any point in the domain R and evaluating on the wave front $\phi(s,n) = 0$, we get

$$n_{\phi\psi} = -\left\{ \frac{\gamma M_0^3 + 2M_0 (M_f^2 + 1)_0}{2c_0 (1 + M_f^2)_0^{1/2} (M^2 - M_f^2 - 1)_0^{3/2}} \right\} \pi_2 \tag{15}$$

Equation (13) provides that

$$q_{\phi\psi} = -C_1 \pi_2 \tag{16}$$

The amplitude $a(s)$ of the weak wave can be defined by

$$a(s) = [q_n] = \pi_2 / n_\phi \tag{17}$$

Differentiating Eq. (17) with respect to s and making use of Eqs. (15) and (16) we obtain

$$\frac{da(s)}{ds} + C_1 a - C_2 a^2 = 0 \tag{18}$$

with

$$C_2 = \frac{\gamma M_0^3 + 2M_0 (M_f^2 + 1)_0}{2c_0 (1 + M_f^2)_0^{1/2} (M^2 - M_f^2 - 1)_0^{3/2}}$$

This is the fundamental growth equation governing the propagation of the wave.

Global Behavior of Waves

The solution of Eq. (18) is of the form

$$a(s) = e^{-C_1 s} \left\{ \frac{1}{a(0)} - \frac{C_2}{C_1} (1 - e^{-C_1 s}) \right\}^{-1} \tag{19}$$

where $a(0)$ is the value of $a(s)$ at $s=0$. The solution in Eq. (19) shows that if $a(0) < C_1/C_2$, the wave amplitude will monotonically decay and tend to zero as $s \to \infty$. On the other hand, if $a(0) > C_1/C_2$, there exists a critical value S_c of S given by

$$S_c = \frac{1}{C_1} \log \left\{ \frac{a(0)}{a(0) - C_1/C_2} \right\} \tag{20}$$

such that the wave amplitude $a(s)$ increases without limit and becomes unbounded at $s = S_c$. Consequently, a weak wave will break down at the stage $s = S_c$ and a shock-type discontinuity will appear automatically. In the case of $a(0)=C_1/C_2$, a weak wave will stabilize as a wave of constant amplitude. From Eq. (20) we have

$$\frac{dS_c}{dC_1}=\frac{1}{C_1}\left\{\frac{\exp(C_1S_c)}{a(0)C_2}-S_c\right\}>0$$

which implies that the effect of radiative heat transfer will delay shock formation.

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References

[1] Becker, E., "Relaxation Effects in Gas Dynamics," *Aeronautical Journal*, Vol. 74, 1970, pp. 736-748.

[2] Bowen, R. M. and Chen, P. J., "Some Comments on the Behavior of Acceleration Waves of Arbitrary Shape," *Journal of Mathematical Physics*, Vol. 13, 1972, pp. 958-960.

[3] Ram, R. and Srinivasan, S., "Propagation of Sonic Waves in Radiating Gases," *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 57, 1977, pp. 191-193.

[4] Jeffrey, A., "The Development of Jump Discontinuities in Nonlinear Hyperbolic Systems of Equations in Two Independent Variables," *Archive for Rational Mechanics and Analysis*, Vol. 14, 1963, pp. 27-37.

[5] Suhubi, E. S. and Jeffrey, A., "Propagation of Weak Discontinuities in a Layered Hyperelastic Half-Space," *Proceedings of the Royal Society of Edinburgh*, Vo. 75, 1976, pp. 209-221.

[6] Varley, E., "Acceleration Fronts in Viscoelastic Materials," *Archive for Rational Mechanics and Analysis*, Vol 19, 1965, pp. 215-225.

[7] Green, W. A., "The Growth of Plane Discontinuities Propagating in a Homogeneously Deformed Materials," *Archive for Rational Mechanics and Analysis*, Vol. 16, 1964, pp. 79-88.

[8] Collins, W. D., "One Dimensional Nonlinear Wave Propagation in Incompressible Elastic Materials," *Quarterly Journal of Mechanics and Applied Mathematics*, Vol. 19, 1966, pp. 259-328.

[9] Ram, R. and Pandey, B. D., "On the Growth and Decay of Acceleration Waves in Transient Gas Flows with Vibrational Relaxation," *Acta Mechanica*, Vol. 33, 1979, pp. 171-178.

[10] Olfe, D. B., *Radiation and Re-entry*, Academic Press, New York, N.Y., 1968.

Single Pulse Laser Irradiation of Fiberglass

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Introduction

THE phenomenology associated with high-power laser irradiation of materials has been of interest for many years.[1-3] This Note will introduce the concept of "residual energy" in a laser irradiated material and examine its effect on the mass removed from a fiberglass surface by pulsed laser radiation. The accumulation of the "residual energy," which is defined as that portion of pulse energy remaining in the material after the termination of the laser pulse, is expected to pyrolyze the resin (with a pyrolysis temperature of approximately 900 K) which binds the plies of glass fibers that form fiberglass. In addition, as the laser pulse fluence increases, the in-depth vaporization of fiberglass limits the magnitude of the residual energy useful for potential multiple pulse mass removal mechanisms.

Analysis

A one-dimensional transient heat conduction analysis with in-depth absorption of laser radiation has been formulated for fiberglass composites, and the governing equation can be written as

$$\rho C_p\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)+(C_p)_r\frac{\partial}{\partial x}[\dot m_r(x)T]+\phi_r\omega_r+\Delta H\omega-\frac{\partial I}{\partial x}\tag{1}$$

where ρ is the density of the solid; C_p is the specific heat of the solid; T is the temperature; K is the thermal conductivity; $(C_p)_r$ is the specific heat of the resin gas; $\dot m_r$ is the mass flux of the resin gas; ϕ_r is the enthalpy of pyrolysis per unit mass of gas generated; ω_r is the rate of gas generation per unit volume; ΔH is the heat of vaporization of solid; ω is the vaporization rate per unit volume; I is the absorbed laser intensity; and t and x are the time and axial coordinates, respectively. For the radiative source term, it is assumed that $I=I_0e^{-\tau}$, where I_0 is the initial laser intensity (at $x=0$); τ is the optical depth defined as[4]

$$\tau=\int_0^x K_\upsilon\,dx$$

and the absoption coefficient K_υ is assumed to scale as the solid density. Equation (1) has been solved by an explicit forward-marching technique in finite difference form with an initial condition $T(x,t=0)=T_0(x)$ and the boundary conditions for $t>0$

$$-K\frac{\partial T}{\partial x}=I_0[1-\exp(-K_\upsilon\Delta x/2)]+(C_p)_r\dot m_rDT+\phi_r\omega_r\Delta x/2-\dot m\Delta H-\epsilon\sigma T^4-\frac{\rho C_p\Delta T}{\Delta t}\frac{\Delta X}{2},\quad x=0$$

$$\frac{\partial T}{\partial x}=0,\quad x=L$$

where $\dot m$ is the vaporization rate at the surface; ϵ is the emissivity; σ is the Stefan-Boltzmann constant; and L is the wall thickness. The pyrolysis rate is adopted from Ref. 5.

$$\omega_r=-\rho_r^3[1.39\times10^{-9}\exp(-20440/T)+11570\exp(-8556/T)]$$

where ρ_r is the resin density. Expressions for ρ_r and $\dot m_r$ are given in Ref. 3.

For surface temperatures well above the normal boiling point, the surface pressure p_s is given approximately by $P_s\approx\rho_\upsilon a^2$, where ρ_υ is the vapor density and a is the sonic velocity of the escaping vapor. But, P_s as a function of T can be approximated by

$$P_s=P_0e^{-\Delta H/RT}$$

where p_0 is a constant which can be determined from the normal boiling point, and R is the ideal gas constant. With the sonic velocity given by $a=\sqrt{\gamma RT}$, where γ is the ratio of specific heats, the vaporization rate at the surface becomes

$$\dot m\approx p_0e^{-\Delta H/RT}/\sqrt{\gamma RT}$$

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